Many Body Operations with Superconducting Circuits

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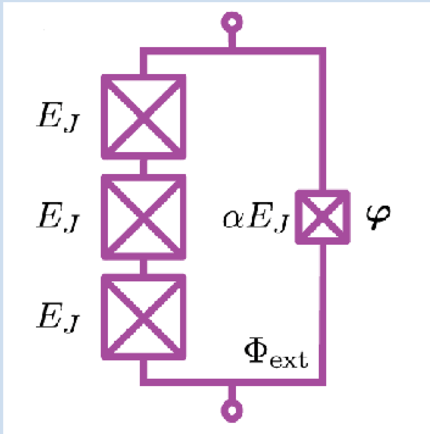
Introduction

Previously three (and multi) qubit gates have been broken down into one and two qubit operations. The effect of many gates acting in series dramatically increases the gate time and decreases the fidelity of the overall gate. Current implementations such as the IBMqx2 have performed Toffoli gates in approximately 20 μs with a fidelity of around 40-50% .

Here we propose a system which can perform three qubit operations in a single step. This approach significantly reduces the gate's error and execution time. Importantly, single shot Toffoli gates are not affected by multiplying gate fidelitys. We use a microwave drive frequency to activate coupled superconducting qubits which perform the gate operation. The drive can be chosen to generate any gate which can be broken down into a series of Pauli operations (for example a CNOT gate).

These qubits are connected in a superconducting circuit and are linked together by a mediator which we chose to be a SNAIL. This SNAIL allows the qubits to be controlled to a high precision with microwave frequencies.

SNAIL



Here E_J is the Josephson energy of the Josephson Junctions, φ is the flux across the smaller junction and Φ_e is the external magnetic flux threaded through the loop.

Simulation Results

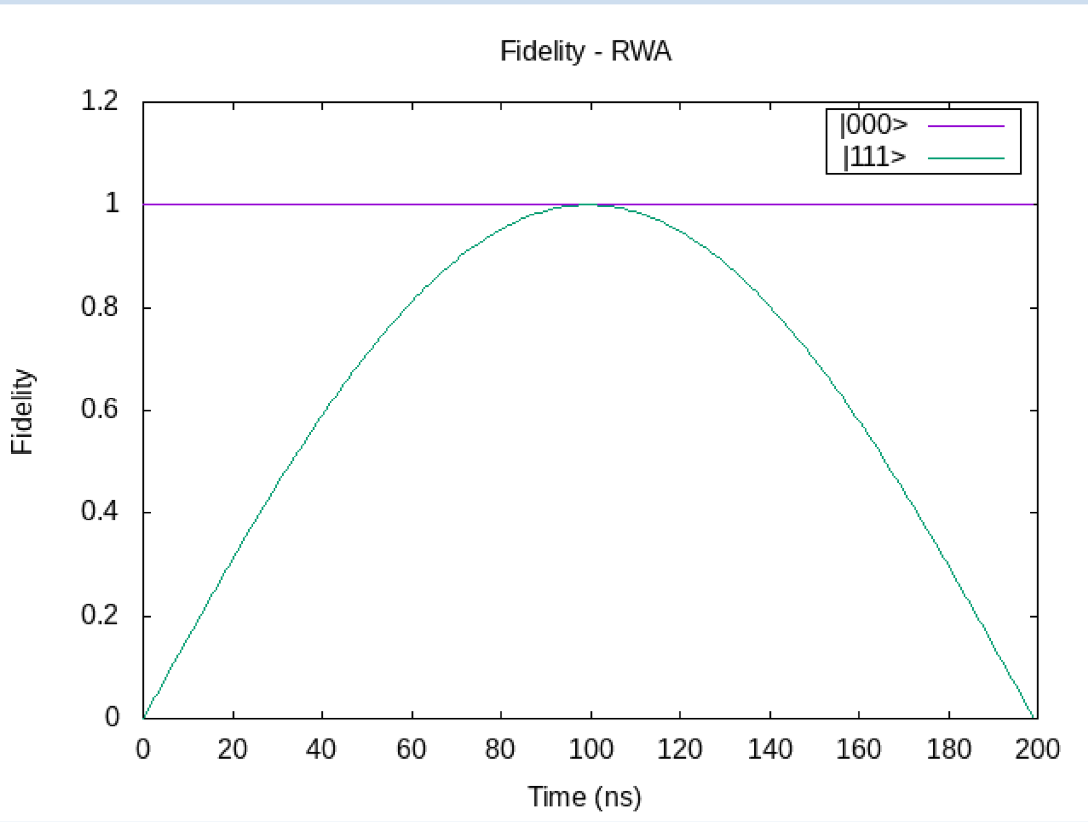


figure 1: Fidelity measurements for two simulations starting with different initial conditions.

I measured the fidelity of the system against target density matricies.It is seen here in the RWA that the system exhibits high fidelity (close to 100%). We measure the fidelity at the first point the lines cross as this gives us the highest average fidelity. Here the gate is an almost perfect Toffoli gate however we must include the Perturbations from other sources to simulate a more physcial system

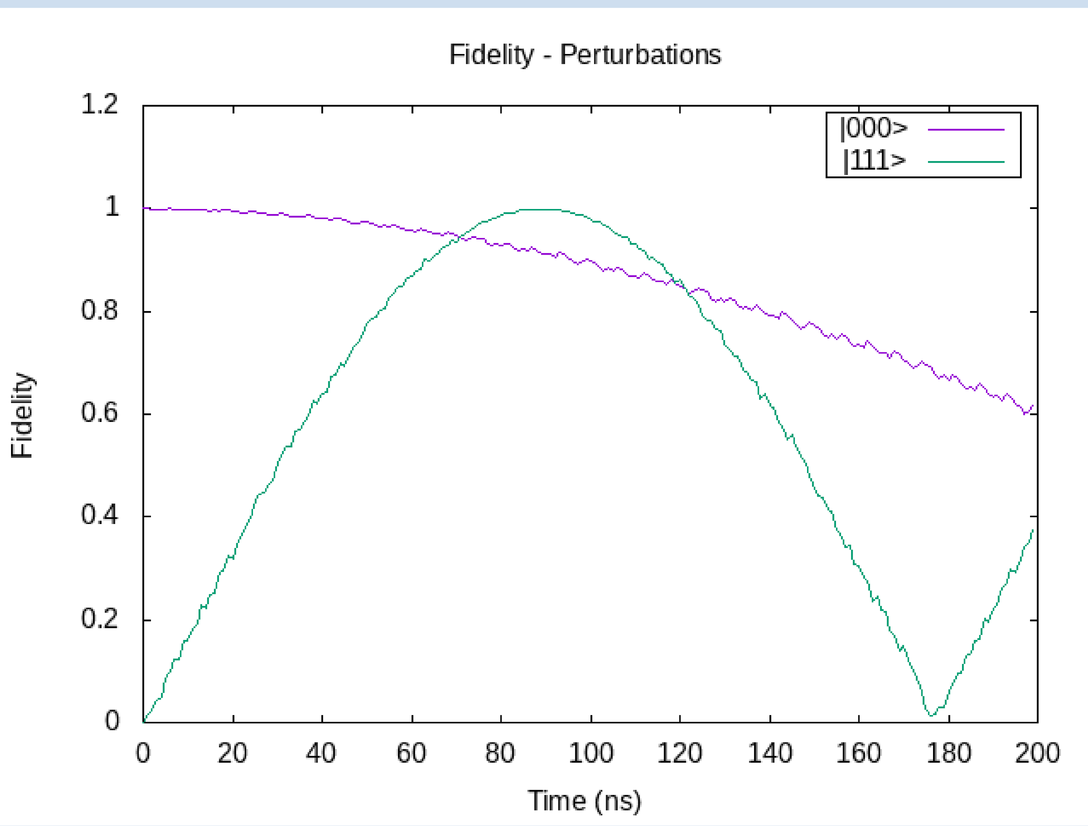


figure 2: Fidelity measurements for a more physical system with the perturbations added in. This produces a Gate Fidelity of around 90% at the previous predicted gate time.

Adding in the perturbations to the system significantly reduces the Gate fidelity and changes the optimim gate time by around 10%. We can account for all of these by calculating the shifts to the drive frequency caused by these perturbations and correcting the drive frequency's accordingly.

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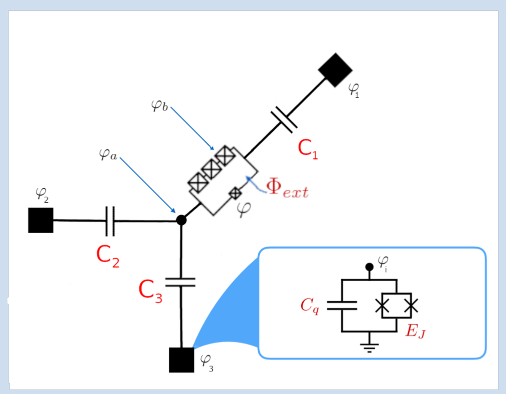
The Circuit

- This circuit is inspired by a circuit designed to create 4 body interactions which originally contained a DC-SQUID. However the DC-SQUID did not produce three body interactions.
- To produce three body interactions we replaced the DC-SQUID with a SNAIL (1) which had the potential

  $$U_{SNAIL} = -\alpha E_J \cos(\varphi) - 3E_J \cos(\frac{\varphi - \varphi_e}{3}),$$

  The nonlinearities of the $\cos(\frac{\varphi-\varphi_e}{3})$ produced the three body interactions which were not present in the DC-SQUID circuit.
- From the full Hamiltonian we eliminate the Qubit-SNAIL interactions using a Schrieffer-Wolff transformation which also bring out the n-body qubit interaction where $\varphi_{q-} = \varphi_1 + \varphi_2 - 2\varphi_3$

  $$H \approx \tilde{E}_J F(t)(\frac{1}{2}\varphi_{q-} + \frac{1}{8}\varphi_{q-}^2 + \frac{1}{48}\varphi_{q-}^3).$$



We can tune the drive frequencies such that the different numerical factors in front of the various interaction terms are canceled out and do not lead to unwanted perturbations.

Time Dependant External Flux

Previously the external flux through these types of circuits has been a uniform magnetic flux, meaning that the magnetic field was time independent. For our purposes however we require that the external flux through the circuit be a time dependant fast oscillating flux. It is the specific shape of this drive that triggers the Toffoli interaction we are seeking.

We must be careful when we deal with the external flux since variable transformations may now produce unwanted derivative terms in the kinetic Energy.

It should also be noted that there is a gauge freedom in the choice of which branch the external flux is associated with. Careful consideration must be made when dealing with this time dependant flux. We follow the suggestions made in (2) to arrive at a new more accurate potential

$$U_{SNAIL} = -3E_J \cos(\frac{\tilde{\varphi} - C_r\varphi_e}{3C_\Sigma}) - \alpha E_J \cos(\frac{\tilde{\varphi} - C_l\varphi_e}{C_\Sigma}).$$

Where C_l and C_r are the capacitance's of the left and right arms, and C_Σ is the sum of all the captaincies in the circuit. This potential still exhibits the three body interactions we are seeking.

Conclusion

This project showed that three body interactions were theoretically possible and we have numerically shown that a Toffoli interaction is indeed possible as well. These numerical results have shown that the fidelity of this gate could be upwards of 99% in a perfect environment which is a dramatic improvement on current fidelity's of Toffoli gates. Some potential other applications of this circuit could be

- Applications in Topological Quantum Error Correction,
- Application in Quantum Simulations of Lattice Field Theory, especially in QED,
- Producing other Quantum Gate such as a CNOT gate.

References

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